Screening methods to reduce complex models of existing structures

Recalculations, reuse or estimations of residual lifetimes are typical examples for evaluations of existing structures with numerical models. The models have to include a variety of scattering parameters. Moreover, they scatter over space and time what usually causes a high computational cost in stochastic analyses that should be thoroughly reviewed for suitable model reductions. Thus, screening methods are helpful to identify those parameters of minor impact to the model's output that may be subsequently neglected. They require less computational cost than general sensitivity analyses. The reason for this is that they analyze the model independently of the actual distribution types of the input parameters. Rather, they selectively screen the model space.

Two examples, namely a damage analysis of a masonry façade exposed to tunneling induced settlements and a lifetime prognosis of a pre-stressed concrete bridge under interactive environmental, physical and chemical actions, demonstrate the application of screening and its impressive benefits. In both cases, reductions in about 2/3 of the initial parameters are achieved with significant savings in computational efforts but no considerable losses of accuracies.

1 Introduction

To evaluate the bearing capacity of existing structures, ever more complex calculation models are being used. For a differentiated and detailed understanding of these models, stochastic analyses are an established method, considering the uncertainty from many model parameters. This often requires a large number of calculations.

Sensitivity analyses (SA) investigate the impact of scattering input parameters on the scatter of a model result. Primary goals are a better understanding of the model behavior and the model reduction [1]. In this context, model reduction means that after the SA has been carried out, input parameters relevant to the results are retained in the model and irrelevant parameters are fixed at a deterministic value, e.g. their mean value. The parameter reduction is suitable for complex models with a large number of input parameters and for models with high computational costs. This is the aim of the screening procedure presented here, which - in contrast to the very time-consuming Sobol’ indices, for example - can essentially be used independently of the distribution shape of the input parameters.

2 Sensitivity analysis

Sensitivities can be determined with a variety of different methods. A comprehensive overview of the individual methods can be found, for example, in [2]. While local SA are based on partial derivatives and only lead to valid results in the immediate vicinity of the selected evaluation point [1], global SA consider the entire value range. The global methods can be distinguished into quantitative and qualitative methods. Quantitative methods analyze the contribution of input variance to the output variance. However, they require many model calculations for reliable results. Qualitative methods (screening methods) estimate sensitivities from only a few calculations. In this paper, the most common screening method [3]-[4], the so-called Elementary Effect method, is used for parameter identification.

2.1 Elementary Effect method

The Elementary Effect method (EE-method) [5] bases on successive variation of all input parameters and the analysis of their effect on the result. A model $Y$ with $k$ independent input parameters $(X_i)_{i=1,\ldots,k}$ is considered, whereby the input parameters are normalized. Thus, they span a $k$-dimensional unit space $\Omega$. This unit space is discretized into a grid with $p$ entries (so-called $p$-level grid). This discretization then specifies how large the increment $\Delta = p/(2p-2)$ should be, by which each parameter can be varied in random order [1]. As soon as each parameter has been varied once, a so-called trajectory of $k+1$ steps results. This trajectory can be imagined as a diagonal sequence of steps running through a space between a starting point and a final point (Fig. 1).

Subsequently, the $i$-th elementary effect $EE_i$ is to be determined by Eqn. (1) for a total of $r$ trajectories. For sufficient coverage of the entire unit space and taking into account the lowest computational effort possible, [1] recommends a grid of $p = 4$ entries and $r = 10$ trajectories.

$$EE_i = \frac{Y(X_1,\ldots,X_i+\Delta,\ldots,X_k) - Y(X_1,\ldots,X_i,\ldots,X_k)}{\Delta}$$ (1)
The Elementary Effects $EE_i$ serve to calculate the sensitivity measures from Eqn. (2) to (4). The mean value $\mu$ and the absolute mean value $\mu^*$ reflect the mean influence of the input parameters. The latter also eliminates misinterpretations due to algebraic signs [3]. The variance $\sigma^2$, on the other hand, determines nonlinear effects and interactions with other parameters.

\[
\begin{align*}
\mu_i &= \frac{1}{r} \sum_{j=1}^{r} EE_i^j \\
\sigma_i^2 &= \frac{1}{r-1} \sum_{j=1}^{r} (EE_i^j - \mu_i)^2 \\
\mu_i^* &= \frac{1}{r} \sum_{j=1}^{r} |EE_i^j|
\end{align*}
\]

Input parameters are randomly generated within their individual boundaries according to Eqn. (5). There $J_{k+1,1}$ is the one-matrix and $J_{k+1,1}$ is the corresponding one-vector. The starting values for generating a trajectory are randomly selected from the interval $\left[0, \frac{1}{(p-1)^2}, \frac{2}{(p-1)^2}, \ldots, 1 - \Delta\right]$ and summarized in the vector $x^\star$. $B_{k+1,k}$ is a lower triangular matrix of ones. The diagonal matrix $D^\star$ corresponds to an identity matrix with a random sign $(+1, -1)$ with the same probability of occurrence and is responsible for the ascent and descent of the individual coordinates of a trajectory. The permutation matrix $P^\star_{k,k}$ varies the order in which the parameters are changed by $\Delta$.

\[
B^\star = (J_{k+1,1} \cdot x^\star + (\Delta/2) \left( (2B_{k+1,k} - J_{k+1,k})D^\star_{k,k} \right) + J_{k+1,1})P^\star_{k,k}
\]

If Eqn. (5) is extended to grouped input, several input parameters are combined into groups when, for example, there is an indication that their influence is irrelevant. Within a group, the grouped parameters vary in different directions, so that the method for determining the elementary effects according to Eqn. (1), in which the function value is expressed at position $X$ by subtracting the function value at $X + \Delta$, cannot be applied. Therefore, it is recommended in [4] using the absolute mean $\mu_i$ instead. In this way, however, it becomes impossible to identify nonlinear correlations or interactions between the input parameters. Investigations in [1], prove that the differences between $\mu_i$ and $\sigma$ are negligible and yet sufficiently precise statements can be made. Only the sample matrix $B^\star$ needs to be adjusted to determine the elementary influences according to Eqn. (6). $G$ corresponds to a group matrix with $k$ rows and $g$ columns, and $g$ also indicates the number of groups. An element $G_{i,j}$ of the group matrix is assigned a value of 1, if the input parameter $X_i$ belongs to a group $j$, otherwise the value is 0. In cases where $g$ equals $k$ each parameter has its own group and thus the sampling procedure yields the same results as the original approach according to Eqn. (5).

\[
B^\star_{gr} = J_{g+1,1} \cdot x^\star + \left( \frac{\Delta}{2} \right) \left( (2B_{g+1,1} (G_{k,g}P^\star_{g,g})^T - J_{g+1,1})D^\star_{k,k} + J_{g+1,1} \right)
\]

In case of an unlimited distribution function, e.g. the Gaussian normal distribution, the ends of the function need to be truncated. The $\pm \infty$-unit space is transformed to the new limits by Eqn. (7), where $B^\star$ is determined according to Eqns. (5) or (6). The upper quantile $Q_u$ and the lower quantile $Q_l$, respectively, are chosen by the user. As a rule, the quantiles at 0.5% and 99.5% can be selected. Subsequently, the evaluation of the inverse cumulative density function $F^{-1}(x)$ provides suitable values for $B^\star_{new}$.

\[
B^\star_{new} = F^{-1} \cdot B^\star \cdot [Q_u - Q_l] + Q_l
\]

### 2.2 Generation of multiple trajectories using distance matrices

Since more than one trajectory should be processed, the equal coverage of the parameter space and the avoidance of similar trajectories are beneficial. In [3], an extension of the procedure based on distance matrices is proposed: A large number ($r_0$) of $B^\star$-matrices (i.e. trajectories) can be generated with little effort. From this large quantity of trajectories, a rather small number $r$ of the most dissimilar ones is selected and used in the analysis. To determine the most dissimilar ones, the distance between two trajectories $d_{ml}$ is determined by:

\[
d_{ml} = \begin{cases} 
\frac{\sum_{i=1}^{q+1} \sum_{j=1}^{q+1} \sum_{x=1}^{q} (x_1^{(i)}(m) - x_2^{(j)}(l))^2}{2} & \text{if } m \neq l \\
0 & \text{else}
\end{cases}
\]

Here $x_1^{(i)}(m)$ is the normalized realization for the parameter $z$ at the $i$-th point of the $m$-th trajectory. The "best" $r$ trajectories are determined by maximizing the distance $d_{ml}$. This
The sampling of the input parameters is carried out afterwards. As in other sampling methods, the realizations of each parameter are generated using the inverse cumulated distribution function (cdf). For the boundary values of the [0, 1]-interval, realizations \( x \rightarrow \pm \infty \) could be generated, depending on the distribution function. However, to avoid this, the boundaries must be cut off or the realizations have to be selected within the subintervals [1].

Using a user-selectable number of trajectories with \( r \geq 2 \) [1], the elementary effect of the parameter \( i \) is determined and following the suggestion in [6] — scaled by the basis of the scatter of the simulation result \( r_y \).

\[
EE_i = \frac{1}{r_y} \frac{f(x'_1; \ldots, x'_r + \Delta \ldots, x'_{r}) - f(x'_1; \ldots, x'_r)}{\Delta} \tag{9}
\]

According to [1] a "meaningful measure" between \( p \) and \( r \) has to be found, since an evaluation of many trajectories \( r \) with a small number of subintervals \( p = (r-1) \) as just as inefficient as vice versa. An odd number \( p = (r-1) \) of subintervals is to be preferred in order to better cover the full range of values. The improved approach for selecting the best paths [3], yields reliable results with \( r = 10 \) and \( p = 4 \) [1], whereby this holds true independently from the number of model parameters.

3 Damage assessment of masonry buildings

Within the framework of urban tunnel alignment design, settlements resulting from the construction and any consequential damage to surface structures must be assessed. A large number of models and methods are available for this purpose. For an overview of such see e.g. [7]; [8]. If the predicted settlements or damages exceed defined limit values, damage-reducing compensation- or support-measures are necessary from the point of view of the design.

Here, the effects of scattering input parameters on the damages to a façade in the vicinity of a new tunnel alignment is determined [9] (Fig. 2, a). The analysis is based on a finite element (FE) model with nonlinear material behavior. The façade (Fig. 2, b) is represented by a center line model with shell elements for the masonry and plate elements for the reinforced concrete foundation. The behavior of the ground and the tunneling-induced settlements are captured by geometrically nonlinear springs at the foundation. Detailed information on the material models for masonry and concrete as well as the spring elements can be found in [8]. The settlement trough is defined by the model of Peck [10] for the transversal direction of the tunnel. Its cross section represents a profile analogous to the Gaussian distribution (Fig. 2, c).

The maximum strain values of the FE calculation are used to determine the EE. Since the damage magnitude is irrelevant for the sensitivity analysis, no further differentiation of the results is necessary.

The simulation model consists of \( k = 21 \) independently scattering input parameters, which are described by uniform- (U), normal- (N) or lognormal-distributions (LN). Tab. 1 shows these parameters. 12 parameters represent the geometry (geom.), 8 the material properties (mat.) and one the load on the façade. Due to variable window sizes, only the scatters around the mean values are given in lines 5 and 6. For simplicity, the same scatter is used for all 32 openings. The materials’ moduli of elasticity are also included in the models. These are fully correlated, i.e. functionally dependent on the sizes given in Tab. 1. More detailed information can be found in [8].

<table>
<thead>
<tr>
<th>parameter</th>
<th>distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>height of the façade [m]</td>
<td>Height (H)</td>
</tr>
<tr>
<td>width of the façade [m]</td>
<td>Width (B)</td>
</tr>
<tr>
<td>width of the foundation [m]</td>
<td>Width (W)</td>
</tr>
<tr>
<td>height of the foundation [m]</td>
<td>Height (H)</td>
</tr>
<tr>
<td>variation of windows' widths [m]</td>
<td>Window width variation (Ww)</td>
</tr>
<tr>
<td>variation of windows’ height [m]</td>
<td>Window height variation (Wh)</td>
</tr>
<tr>
<td>concrete cover [m]</td>
<td>Concrete cover (Cc)</td>
</tr>
<tr>
<td>bending reinforcement [cm²/m]</td>
<td>Bending reinforcement (Br)</td>
</tr>
<tr>
<td>shear reinforcement [cm²/m]</td>
<td>Shear reinforcement (Sr)</td>
</tr>
<tr>
<td>compr. strength of masonry [N/mm²]</td>
<td>Compressive strength (f_cm)</td>
</tr>
<tr>
<td>tensile strength of masonry [N/mm²]</td>
<td>Tensile strength (f_tm)</td>
</tr>
<tr>
<td>concrete's compr. strength [N/mm²]</td>
<td>Concrete's compressive strength (f_c)</td>
</tr>
<tr>
<td>yield limit of steel (Stahl) [N/mm²]</td>
<td>Yield limit of steel (f_y)</td>
</tr>
<tr>
<td>Young’s mod. of steel (Stahl) [N/mm²]</td>
<td>Young's modulus of steel (E_s)</td>
</tr>
<tr>
<td>load [kN/m]</td>
<td>Load (L)</td>
</tr>
<tr>
<td>coverage [m]</td>
<td>Cover (C)</td>
</tr>
<tr>
<td>outer diameter of the tunnel [m]</td>
<td>Outer diameter (D)</td>
</tr>
<tr>
<td>eccentricity [m]</td>
<td>Eccentricity (E)</td>
</tr>
<tr>
<td>rigid modulus of the soil [MN/m²]</td>
<td>Rigid modulus (E_s)</td>
</tr>
<tr>
<td>Poisson’s ratio [ ]</td>
<td>Poisson’s ratio (ν)</td>
</tr>
<tr>
<td>dimensionless soil parameter[-]</td>
<td>Dimensionless parameter (K)</td>
</tr>
</tbody>
</table>
To calculate the EE for the façade $r = 4$ trajectories are selected. Thus, a total of $(k + 1)r = 88$ FE simulations are required. The edges of the unit cube are divided into $p = 1 = 3$ subintervals. For non-equally distributed parameters the limit values of the distribution functions are selected to 1.35% and 98.65% quantiles, in order to obtain reliable sensitivities and to prevent false predictions due to low limit values. A 5% significance level determines whether a parameter is of relevant or irrelevant influence. It refers to the absolute mean value of the EE and is derived from Eqn. (10).

$$E_i = \frac{\mu_i}{\sum_{i=1}^{k} \mu_i^2} \text{ für } i = 1, ..., k$$

Fig. 3 shows the resulting EE for the simulation model. Blue bars represent significant parameters; those without significant influence are represented by orange bars, like the effects of all geometric parameters for example. Their impact on the output scatter is negligible and hence they can be fixed to deterministic values. Therefore, for subsequent analyses only 6 scattering material inputs have to be considered. These are the concrete compressive strength $f_{cm}$, the masonry compressive strength $f_{cm}$ and tensile strength $f_{int}$ for the façade, the rigid modulus of the soil $E_p$, the Poisson’s ratio $v$ as well as the dimensionless soil parameter $K$. The remaining 15 parameters are fixed to their mean values.

4 Assessment of pre-stressed concrete bridges

To analyze the accuracy of lifetime predictions of prestressed concrete bridges subjected to fatigue, a stochastic calculation model consisting of 23 basic variables was developed as part of a DFG (German research foundation) research project. The model output is the fatigue damage $D$ of the pre-stressing steel after 200 years of service lifetime.

The reference structure for the prognosis model is a flyover in Düsseldorf, Germany, the so called "Pariser Straße" (Fig. 4). This 13-span pre-stressed concrete box girder bridge was built in 1959/60 and dismantled after more than 50 years of service life; details on the structure can be found in [11].

4.1 Lifetime-prediction model for fatigue

The prognosis model consists of a FE calculation of the structure for the determination of internal forces and a separated stress calculation on cross-section level, cf. [11]. In addition, results from temperature simulations ($\Delta T$), data from traffic counts ($dn/dt$) and structural measurements are incorporated to the model.

Transient loadings are generated by the fatigue load model 4 (FLM 4) and by vertical temperature gradients as restraints. The latter are included as a distribution according to the German recalculation guideline, stage 2; here only the decisive lower edge of the distribution [-6 K; -8 K] (Fig. 4). The scatter of the relative frequencies is considered according to an approach in [15]. The influence of variable axle loads of heavy trucks is described by factors $(N(1, 0.12))$, which scale the loads of the five truck types of FLM 4 (Fig. 4). The five types differ in the axle numbers and loads according to Eurocode 1-2:2010.

The superposition of these loads results in upper and lower internal forces. Thus, stresses in the pre-stressing steel and subsequently stress ranges $\Delta \sigma$ can be determined. These ranges serve for fatigue calculations by means of the Pålmgren-Miner-hypothesis using $S$-$N$-curves $(k, \Delta \sigma (N^*))$ (Fig. 4); the stress ranges remain within the limits of high-cycle fatigue. Therefore, the inclination in the low-cycle fatigue is set deterministically to $k_1 = 3.0$. The width of the deck slab $b_{DS}$ is the only variable of the cross-section’s geometry considered; it even covers any variances of other geometric variables, cf. [12].
4.2 Application of the EE method

The EE method serves to identify the less relevant model parameters of the prediction models with the aim to reduce the number of variables in the stochastic calculations as shown in [15]. For further model analyses, Sobol’ indices [16] are used.

All 23 independent input parameters are given in Tab. 2. They are either normally (N) or lognormally (LN) distributed. For the screening, the range of their realizations is defined by the interval $[μ - 3σ; μ + 3σ]$ and the $0.1\%$ and $99.8\%$ quantiles, respectively. The 23 ranges of values span a space of 23 dimensions. Using the normalization of the space to a (23-dimensional) unit space $Ω [0, 1]$, the increment $Δ \geq 0.5$ by which each parameter is increased or reduced is the same for all $[1, 5]$. $Ω$ is divided into $p - 1 = 6$ subintervals. The increment results from Sect. 2.1 to $Δ = 0.5B33$; the interval width yields $1/(p - 1) = 0.1667$.

Using different trajectories with variable step sequences, different model regions can be included in the evaluation and the space from one corner $x^{(t)}$ to the quasi opposite $x^{(t+1)}$ is crossed. These paths are generated in advance using distance matrices. Doing so, the $r = 10$ most unsimilar sample matrices are selected from $n_0 = 1000$ generated matrices $B^*$. The individual matrices $B^*$ differ in:

- the sequence in which the parameters are changed $→ q^*$ combinations
- the sub-interval from which the first realization for the starting point $x^{(t)}$ is selected $→ q^*$ combinations
- the decision as to whether to increase or decrease the parameter $i$ by $Δ$.

A total of $\binom{1000}{2} = 499500$ distances between the matrices are determined according to Eqn. (8). The combination of the $r = 10$ largest distances is then used to calculate the $EE$.

4.3 Results

The screening result of the prediction model is depicted in Fig. 5. Blue colors indicate relevance, orange colors indicate negligible significance. The specific value of each influence factor $(μ, σ, μ^*)$ is less important due to its independence from the distribution shape; in fact, the qualitative order is more significant.

Fig. 5 EE of the nonlinear FE-simulation for 23 input parameters

The extents of $μ, σ$ and $μ^*$ almost comply for most parameter giving rise to columns of comparable heights. Quantitatively speaking, the decisive parameters are the losses in the pre-stressing force $α_{CS}$, the vertical position of the pre-stressing steel $z^*_v$, and the truck type no. 3 of FLM 4 represented by $w_3$. Those parameters with $μ < 0.02$ and $σ < 0.04$ are interpreted here to be of minor relevance. These are, amongst others, the concrete compressive strengths $f_c$, and the truck-types no. 1, 2, 4 and 5 from FLM 4. Since stresses in the pre-stressing steel under fatigue load generally remain within the elastic range, the $EE$ for the yield strength $f_{y0.1}$ is omitted.

With these results, the model can be reduced from 23 to 8 significant variables (blue bars) by fixing the other 15 variables (orange bars) to their specific mean values. For this purpose, a rather small amount of $\binom{k + 1}{r} = (23 + 1) \cdot 10 = 240$ calculations were necessary.

5 Conclusions

The contribution shows an efficient way to reduce complex models with numerous variable and scattering parameters using the EE-method. As a result, variables with little influence on the stochastic calculation result can be fixed to deterministic values (factor fixing), usually to their mean value. In comparison to other sensitivity analysis methods, this requires only a few model calculations.

By the example of the settlement investigations of masonry.
façades it could be shown that scattering values of all geometric parameters can be dispensed with. Rather, the material parameters of the façade as well as the characteristic soil parameters are relevant for the results.

In case of the prediction model of the fatigue damage of a pre-stressed concrete bridge, 15 parameters could be identified as less relevant. Relevant parameters are the variables of the pre-stressing as well as traffic and temperature loads. Focus should be predominantly lead on such parameters, e. g. in the course of a monitoring to achieve data for residual lifetime estimations.

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