NUMERICAL PROCEDURE FOR LONG-TERM DEFLECTION PREDICTION OF RC ELEMENTS SUBJECTED TO DIFFERENT LOAD HISTORIES

SUMMARY

The influence of expected but idealized load histories typical for structural service conditions during life-time was studied with a special emphasis on long-term serviceability of RC structures. For that purpose, a numerical step-by-step cross-sectional procedure based on the Age-Adjusted Effective Modulus Method (AAEMM) has been developed and successfully verified through existing experimental results applying it on simply supported RC beams and one-way slabs. First results confirmed that individual load histories and precisely tracking the stiffness reduction with time, are significant to realistically reflect the long-term behaviour of RC elements.

Keywords: step-by-step, load history, long-term deflections, moment-curvature relations

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1. INTRODUCTION

Concrete creep is traditionally defined to be a kind of time-dependent behaviour exhibited by concrete under sustained constant loads. Experimental studies conducted in the last fifteen years at FCE Skopje [Markovski 2003; Arangjelovski 2011; Nakov 2014] organised to check this traditional view, indicate that the concrete creep does not only appeared under constant sustained load. Results from these studies clearly show that the intensity [Markovski 2003], as well as the duration [Arangjelovski 2011] of the variable loads can significantly influence the time-dependent behaviour of reinforced concrete (RC) structures.

Having this in mind, it is apparent that at certain RC structures, especially those with more intensive and longer term live loads, improper treatment of these loads will lead to underestimation of the long-term concrete effects and consequently to reduced or disrupted serviceability of a structure. In this context, bridges are structures that are most sensitive on creep under variable loads due to the character of the traffic loads. Besides bridges, this kind of load can appear at other types of reinforced concrete structures such as multi-store parking garages, multi-store warehouses and crane beams.

Although current building codes recognize this phenomenological concrete behaviour to creep under variable loads, they offer overly simplistic approach for taking this effect into account in serviceability calculations (Fig. 1). Namely, quasi-permanent load, consisting of all permanent and a fraction of live loads (Fig. 1), is likely to be exceeded during the life-time of the building and mostly it does not provide an insight into the real load history of a considering structure or an element [Criel et al. 2014].

![Fig. 1: Simplification of the variable loads effect in the current building codes](image)

Insisting on more exact prediction of the long-term behaviour of concrete would mean considering a real load history instead of a quasi-permanent combination only. On one hand, consideration of the real load history means knowing and considering the intensity, average number and average duration of the cycles of loading and unloading repeating over one year. On the other hand, deflection sensitive elements like RC slabs in multi-story buildings are commonly subjected to short-term load peaks arising either during construction or during structures service life. While the quasi-permanent value is influenced by the duration and intensity of the live loads, it is completely independent of these short-term load peaks.

Thus, the influence of expected but idealized load histories typical for structural service conditions during life-time was studied here with a special emphasis on long-term serviceability. For that purpose, a numerical step-by-step cross-sectional procedure based on the Age-Adjusted Effective Modulus method (AAEMM) has been developed and successfully verified applying it on simply supported RC beams and one-way slabs. Separately, two characteristic load histories were analysed, one typical for construction phase of a structure and other typical for its service life. Within the considered load histories, many parameters were varied, such as duration of the cycles of loading and unloading, the permanent-to-cracking load ratio and the intensity of the initial load peak during construction.

First results indicated that even short-term, the initial load during construction affects the long-term final deformations due to collateral cracking. Results from the cyclic load histories showed that the permanent-to-cracking load ratio \( \frac{G}{F_{cr}} \), as well as the duration of the loading and unloading cycles
(Δt_c/Δt_{G,Q}=8h/16h and Δt_c/Δt_{G,Q} =24h/24h) influence the long-term final deflections too, but in different ways. Apparently, these characteristics of the load histories are of relevance when assessing long-term serviceability of creep sensitive structures.

2. IMPORTANCE OF LONG-TERM DEFLECTION PREDICTION DUE TO LOAD HISTORIES

The complex and hardly known load history as well as the highly variable concrete long-term properties, represent the main difficulties for predicting the deformations of RC structural members [Burns 2011]. For reasons of simplicity, code-based approaches to deformation prediction usually neglect the unknown load history, thus, serviceability calculations are based on the sustained load case only, or more precisely, on the quasi-permanent load combination proposed by many current codes, e.g. Eurocode 2 [EN 1992-1-1 2004].

This load combination is independent from the actual load history and it is not able to account for the intermittent high live loads or the preloading at young concrete ages. On one hand, high values of live loads are assumed to be temporary and to have little effect on creep deformations. However, although short-term, these temporary load peaks are sufficiently intense and thus indirectly affect the long-term deflections through permanent reduction of element stiffness. This is also confirmed by analysis of data from Cardington [Hossain and Vollum 2002; Vollum et al. 2002] suggesting that final deflections are governed by the most severe cracking induced either by possible construction overload or by peak service load [The Concrete Society 2005].

In addition to the reduced stiffness of the section, cracking also influences the concrete creep and shrinkage deformations since only the uncracked part of concrete is affected by these phenomena. Since the degree of cracking strongly influences the final long-term deflections, these facts indicate that real load histories have to be used instead of quasi-permanent loads.

<table>
<thead>
<tr>
<th>Type of structure</th>
<th>Real load history</th>
<th>Simplified history</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction (1)/ Service life (2)</td>
<td><img src="image1" alt="Real Load History" /></td>
<td><img src="image2" alt="Simplified History" /></td>
</tr>
</tbody>
</table>

Fig. 2: Real vs. simplified load histories for different types of RC structures during construction and service period

By step-wise calculation and sufficiently sophisticated models for taking into account the time-dependent concrete effects, it would be possible to analyse the long-term behaviour of RC elements with regard to load variations over a period of time. However, this is seldom of any practical interest, since the real variation of the loads during the future service life cannot be predicted due to their randomness [Westerberg 2008]. Instead, simplified and idealized expected load histories, as those presented in Fig.2, are suggested to be used in the long-term analysis of the creep sensitive RC structures.
In practice, it is more convenient to calculate deflections corresponding to load histories such as that in Fig.2 in a single step procedure in conjunction with a suitably modified concrete tensile strength, modulus of elasticity and creep coefficient.

Here, the first effort in modelling the behaviour of RC flexural elements considering the complete simplified load histories is presented along with the theoretical background described in the following section.

### 3. NUMERICAL MODELING APPROACH

Numerical analyses for prediction of long-term deflection caused by load histories require application of step-by-step numerical procedures through time discretisation, which is in direct correlation with the considered load history (Fig.3). Subjected to any complex load history, under the assumption of linear behaviour, the total response at any time and at any point of the RC element can be determined by superimposing the individual responses due to the individual load increments (Fig.3) [Khor et al. 2001].

In order to accurately define the effective concrete stiffness and the time-dependent parameters in each time step, it is also necessary to discretise the element relating its cross-sectional height and length. Below, expressions used for prediction of time-dependent local (strains and curvatures) and global (deflections) deformations under time-varying load history (Fig.3) are presented.

#### 3.1 Modelling at cross-sectional level

Determination of the long-term deflections of a reinforced-concrete member requires knowledge of strains, stresses and curvature along the member. These quantities are all time-dependent as a result of the combined effect of creep, shrinkage and concrete maturation under a time-varying load. Herein, in each time step, creep coefficients, shrinkage strains and modulus of elasticity of concrete were calculated according to the Model Code 2010 provisions [fib Model Code 2013].

Based on the principle of superposition, the total curvature can be obtained as the sum of the curvatures caused by load variations (elastic or instantaneous curvature) and the curvatures caused by the time-dependent effects (creep and shrinkage curvature) (Eq. 1) [Khor et al. 2001].

\[
k = \sum_i k_{\text{elas}}(t_i) + \sum_i k_{\text{creep}}(t, t_i) + k_{\text{shr}}(t, t_0)
\]

(1)

The instantaneous curvature (Eq.2) at a certain concrete age \( t \) can be determined by [Khor et al. 2001]:

\[
k_{\text{elas}}(t) = \frac{A(t)M(t) + B(t)N(t)}{E_c(t)[A(t)I(t) - B(t)^2]}
\]

(2)

Where \( A(t), B(t), I(t) \) are the area, the first and the second moment of the transformed cross-section at certain time \( t \), all taken about the randomly chosen, but constant fibre of the cross-section.

The effects of creep and shrinkage, as well as creep recovery are treated as equivalent fictitious forces that would restrain their deformations. These forces are then applied on the transformed cross-section to determine the time-dependent curvatures \( k_{\text{creep}}, k_{\text{rev}} \) and \( k_{\text{sh}} \).
The creep-induced curvature \( k_{\text{creep}}(t_i, t_f) \) produced by load increment \( i \) for time interval \((t_i, t_f)\) can be evaluated using the Age-Adjusted Effective Modulus (AAEM) method as follows [Khor et al. 2001]:

\[
k_{\text{creep}}(t_i, t_f) = \frac{A_e(t_i)\Delta M_{\text{creep}}(t_i, t_f) + B_e(t_i)\Delta N_{\text{creep}}(t_i, t_f)}{E_e(t_i)[A_e(t_i)I_e(t_i) - B_e^2(t_i) - \Delta]}
\]  

(3)

The age-adjusted effective modulus which account for creep during the time interval \((t_i, t_f)\) can be obtained using the Eq. 4:

\[
\overline{E}_e(t_i, t_f) = \frac{E_e(t_i)}{1 + \chi(t_i)\phi(t_i)}
\]

(4)

Where \( \chi(t_i) \) is the time-dependent concrete aging coefficient, for which a value of 0.8 is mostly assumed; \( \phi(t_i) \) is the creep coefficient for the time interval \((t_i, t_f)\); \( A_e(t_i) \), \( B_e(t_i) \) and \( I_e(t_i) \) are the area, the first and the second moment of the age-adjusted transformed cross-section for the time interval \((t_i, t_f)\); and \( E_e(t_i) \) is the modulus of elasticity of the concrete at age \( t_i \).

The equivalent moment \( \Delta M_{\text{creep}} \) and axial force \( \Delta N_{\text{creep}} \) for the creep deformation caused by a load increment at time \( t_i \) can be calculated as [Khor et al. 2001]:

\[
\Delta M_{\text{creep}}(t_i, t_f) = \overline{E}_e(t_i, t_f)\phi(t_i)\left[A_e(t_i)\epsilon_{\text{elas}}(t_i) - B_e(t_i)k_{\text{elas}}(t_i)\right]
\]

(5)

\[
\Delta N_{\text{creep}}(t_i, t_f) = \overline{E}_e(t_i, t_f)\phi(t_i)\left[-B_e(t_i)\epsilon_{\text{elas}}(t_i) + I_e(t_i)k_{\text{elas}}(t_i)\right]
\]

(6)

Where \( A_e(t_i) \), \( B_e(t_i) \) and \( I_e(t_i) \) are the area, the first and the second moments of the uncracked concrete cross-section (neglecting steel portions), since creep occurs only in the uncracked part of the concrete.

In this study, the creep recovery due to unloading was taken into account in a very simple manner. A load removal was treated as a negative load which would produce creep equal, but opposite to that caused by a positive load of the same intensity. This meant that the same creep functions were employed for both loading and unloading phase, which certainly led to underestimation of the final deflections.

The shrinkage-induced curvature \( k_{\text{sh}}(t, t_0) \) can be calculated as [Khor et al. 2001]:

\[
k_{\text{sh}}(t, t_0) = \frac{\overline{A}_e(t, t_0)\Delta M_{\text{shr}}(t, t_0) + \overline{B}_e(t, t_0)\Delta N_{\text{shr}}(t, t_0)}{\overline{E}_e(t, t_0)[\overline{A}_e(t, t_0)\overline{I}_e(t, t_0) - \overline{B}_e^2(t, t_0)]}
\]

(7)

Where \( \overline{A}_e(t, t_0) \), \( \overline{B}_e(t, t_0) \) and \( \overline{I}_e(t, t_0) \) are the area, the first and the second moments of the age-adjusted transformed cross-section for time interval \((t, t_0)\), respectively, where \( t_0 \) is the age of concrete when shrinkage starts to act (in most practical cases, prior to the first loading).

The fictitious moment \( \Delta M_{\text{shr}} \) and axial force \( \Delta N_{\text{shr}} \) that result from the shrinkage effects in time interval \((t, t_0)\) are given by [Khor et al. 2001]:

\[
\Delta M_{\text{shr}} = \overline{E}_e(t, t_0)\varepsilon_{\text{shr}}(t, t_0)B_e(t_0)
\]

(8)

\[
\Delta N_{\text{shr}} = \overline{E}_e(t, t_0)\varepsilon_{\text{shr}}(t, t_0)A_e(t_0)
\]

(9)

Where \( \varepsilon_{\text{shr}}(t, t_0) \) describes the shrinkage strain of concrete for time interval \((t, t_0)\).
3.2 Modelling at member level

The described calculation procedure for the total curvature is repeated for every predefined cross-section along the element and for every time step. Moment-curvature relations can be applied to determine the long-term deflections (Fig.4). The general way is to integrate the total curvatures (instantaneous plus time-dependent) in the predefined cross-sections along the element length, by applying the principles of virtual work and numerical integration [Docevska et al. 2016]:

\[
a = \int_0^l k(x) \cdot \bar{M}(x) \cdot dx
\]

(10)

Fig. 4: Curvature distribution in a partially cracked beams (left) and moment-curvature relations (right)

When long-term deflections of RC cracked elements are analysed, the procedure for total curvature calculation has to be performed twice, once with uncracked section properties (denoted as curvature \( k_t \)) and once assuming a fully cracked section (denoted as curvature \( k_{II} \)). However, the real behavior of a cracked section will be exhibited between these two scenarios as a result of tension concrete contribution between two adjacent cracks (tension-stiffening effect). Therefore, interpolation between both states is necessary.

Variable stiffness due to cracking, as well as tension-stiffening effect can be considered at section level using the mean curvature approach proposed in Eurocode 2 [EN 1992-1-1 2004] and Model Code 2010 [fib Model Code 2013] (Eq. 11).

\[
k_m(t) = \beta \cdot \left( \frac{M_{cr}}{M_a} \right)^2 \cdot k_t(t) + \left( 1 - \beta \cdot \left( \frac{M_{cr}}{M_a} \right)^2 \right) \cdot k_{II}(t) = (1 - \zeta) \cdot k_t(t) + \zeta \cdot k_{II}(t)
\]

(11)

Where \( M_{cr} \) is the cracking moment, \( M_a \) is the applied moment, \( \beta \) is the coefficient that takes the load duration into account (\( \beta = 1 \) for short-term load and \( \beta = 0.5 \) for long-term/repeated load) and \( \zeta \) is the interpolation coefficient.

4. VERIFICATION

The procedure described was applied on the RC flexural elements subjected to typical load histories (Fig.5) to confirm its validity through comparison with the existing experimental results. To verify the validity of the model for cyclic load histories typical for structure service life, experimental results of beam elements (Fig.5) tested at the FCE in Skopje, R. Macedonia [Arangjelovski 2011] were used. On the other hand, experimental results of one-way slabs (Fig.5) taken from the literature [Vollum and Afshar 2009] were recalculated to verify the validity of the model for construction load histories.
The beam and one-way slab were discretised into 28 and 33 elements along their length, respectively. Layers with 10mm thickness were employed over their cross-sectional height to better trace the position of the neutral axis and the change in the cracked portions over time. The time was also discretised into a sufficient number of time steps adapting to the considered load history (Fig.5). The applied step-by-step technique required a large amount of input parameters and separate storing of concrete stresses and strains in each time step. For these purposes, MATLAB software was used.

Fig. 6: Comparison between experimental data and numerical results for long-term deflections of beams (left) and one-way slabs (right) subjected to different load histories

Fig. 6 (left) presents the development of the calculated long-term deflections of beams obtained by use of both possible approaches, namely, using complete cyclic load history and using quasi-permanent load with experimentally established value for the coefficient $\psi_2 = 0.50$. The calculated results show a trend of underestimation of the deflections with time. Generally, they are in good agreement with the experimental data.

Results presented in Fig.6 (right) also show a good agreement with the experimental data obtained on one-way slabs subjected to simplified construction load history and quasi-permanent load only (Fig.5).
5. RESULTS AND DISCUSSION

The calculation procedure was applied to study the time-dependent behaviour of RC elements (Fig.5) under various load histories. Namely, two characteristic load histories were analysed, one typical for construction phase of a structure and other typical for its service life.

Load histories typical for construction phase consisted of an initial load during construction applied at early concrete ages followed by a quasi-permanent load which covers self-weight (sw) and certain live load peaks. Within these load histories, the initial load peak during construction was varied in order to study its influence on the final long-term behaviour (Fig.7).

Those load histories typical for structures service life consisted of continuously repeated loading and partial unloading cycles (Fig.7). The variable load amplitude was constant over the whole loading process and the unloading was always at a permanent load level. Regarding the level of the permanent load in terms of the cracking load, the analysed load histories were generally divided into two main groups. The two investigated portions of the permanent load corresponded to 0.8F<sub>cr</sub> (Group I) and 1.20F<sub>cr</sub> (Group II) (F<sub>cr</sub> is the load causing the first crack), while the total amount of the load remained equal in both groups. The total permanent plus variable load corresponded to the serviceability load, or more precisely to 55% with respect to the ultimate load (0.55F<sub>u</sub>). In addition, in each of both groups, different duration of the loading and unloading cycles were considered, namely, 16h under total load and 8h under permanent load only (∆t<sub>L</sub>/∆t<sub>U</sub>=16/8h), and 24h under total load and 24h under permanent load (∆t<sub>L</sub>/∆t<sub>U</sub>=24/24h). The durations of the loading and unloading cycles were chosen to represent the real duration of the live loads on specific types of RC structures (for example, city bridges, warehouses, parking garages, etc.). Details on characteristic variables for each of the load histories are contained in Fig. 7 below.

<table>
<thead>
<tr>
<th>Construction/Service life</th>
<th>Simplified history</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td></td>
<td>S4: Peak=quasi-perm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S5: Peak=1.4*quasi-perm.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S6: Peak=sw</td>
</tr>
<tr>
<td>Service life</td>
<td></td>
<td>G/F&lt;sub&gt;cr&lt;/sub&gt;=0.80-Group I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G/F&lt;sub&gt;cr&lt;/sub&gt;=1.20-Group II</td>
</tr>
<tr>
<td></td>
<td></td>
<td>∆t&lt;sub&gt;L&lt;/sub&gt;/∆t&lt;sub&gt;U&lt;/sub&gt;=16/8h</td>
</tr>
<tr>
<td></td>
<td></td>
<td>∆t&lt;sub&gt;L&lt;/sub&gt;/∆t&lt;sub&gt;U&lt;/sub&gt;=24/24h</td>
</tr>
</tbody>
</table>

Fig. 7: Analysed load histories

Some results by means of calculated local (curvature) long-term deformations caused by the idealized load histories typical for construction life are shown in Fig. 8. The results indicate that the initial load during construction affects the long-term final deformations due to collateral cracking. Even in short-term, these load peaks indirectly influence the long-term behaviour of RC elements through an initial but permanent reduction of the member’s stiffness, especially when the load intensity exceeds the subsequent load level (in this case a quasi-permanent one).

Results in Fig.8 show that for the construction load peak equal with the following quasi-permanent load, the final long-term curvature is underestimated for 11.33%. This underestimation is even more pronounced (22%) when the construction load peak is for 40% higher than the following quasi-permanent load.
Figs. 9 and 10 show the long-term deflections (denoted as I) caused by the service load histories presented in Fig. 7 for a period of one year. For the purpose of comparison, long-term deflections caused only by the permanent load G (denoted as II) are additionally presented on the same diagram. Due to the concrete phenomenon to creep under repeated variable loads, the final long-term deflections caused by load histories are certainly much higher than those caused by permanent load only. A great part of these differences occurs due to the irreversibility of the deflections appeared even after the first unloading cycle. Such differences are slightly less pronounced in the second case (Group II), where the chosen level of permanent load within the load history is above the cracking load level. This confirmed again the influence of the degree of cracking on the development and final values of the element’s long-term deflections.

Fig. 9: Long-term deflections caused by the load history $\Delta t_G/\Delta t_G+Q = 8h/16h$ (I) and the permanent load only (II) for Group I (left) and Group II (right)

Fig. 10: Long-term deflections caused by the load history $\Delta t_G/\Delta t_G+Q = 24h/24h$ (I) and the permanent load only (II) for Group I (left) and Group II (right)

Fig. 11 shows the influence of the permanent-to-cracking load ratio on the long-term deflections for each time duration of loading/unloading cycles (16h/8h; 24h/24h). Results presented as normalized deflections along time indicate that both varied aspects within the cyclic loading history, namely variable load intensity and duration, influence the development and final value of the long-term deflections.
Regardless the higher final deflection in the case of Group II (which is unsurprisingly due to the higher unloading level), a higher long-term deflection in both cases (16h/8h; 24h/24h) has been noticed in the case of Group I (13%-21%).

The long-term deflections under 16h of loading and 8h of unloading are 8.7% (Group I) and 3.95% (Group II) higher than those caused by the load histories with equal total duration of the loading and unloading phase (24h/24h). These results suggest that the participation of the variable load duration in the load histories may differently influence the long-term deflections, although these effects are relatively small for the considered load histories.

![Fig. 11: Comparison between long-term normalized deflections due to load histories from Group I (grey) and Group II (black) for considered time histories: 16h/8h (left) and 24h/24h (right)](image)

The analysis of the results shows that the differences caused by different variable load intensity are more pronounced than those arising from different duration of loading/unloading cycles. Regarding the considered simplified load histories, this points to higher sensitivity of long-term deflections to intensity than to duration of the variable load.

6. CONCLUSIONS

The time-dependent behaviour of RC flexural elements has been numerically investigated under various load histories. Based on the current results from the numerical modelling, the following conclusions are drawn:

- Having in mind the interrelation between the cracking extend and the final long-term deflection, it is apparent that the quasi-permanent simplified approach will yield inaccurate results in some special cases. This is particularly true when the quasi-permanent level is below the cracking level or when the load history is complex consisting of high construction or live load peaks. This can be also concluded from the presented results in the paper. Therefore, as a solution for this issue, it is suggested that expected or rather real load histories should be used for deflection prediction in order to determine the most unfavourable crack pattern.

- For the cyclic load histories, the permanent-to-cracking load ratio (G/Fcr), as well as the duration of the loading and unloading cycles (ΔtG/ΔtG+Q =8h/16h and ΔtG/ΔtG+Q=24h/24h) influence the long-term final deflections in different ways. For the considered load histories, the differences in long-term final deflections due to the different permanent-to-cracking load ratio are in the range of about 13% to 21%, while those due to different duration of the loading and unloading cycles are among 4% and 9%. Apparently, these characteristics of the load histories are of relevance when assessing long-term serviceability of creep sensitive structures.
REFERENCES


